SL Paper 2 Mock A 2020 - WORKED SOLUTIONS v1

Section A

1. (a) 100 students

(b) $Q_1 = \frac{n}{4} = \frac{800}{4} = 200$, $Q_3 = \frac{3n}{4} = 3 \cdot \frac{800}{4} = 600$

$a = \text{mark}(Q_1) = \text{mark}(200) = 55$

$b = \text{mark}(Q_3) = \text{mark}(600) = 75$

Hence, $a = 55,\ b = 75$

2. (a) Value after 1 year $= 3000 \times 1.046$

Value after 2 years $= (3000 \times 1.046) \times 1.046 = 3000 \times 1.046^2$

Value after n years $= 3000 \times 1.046^n$

Thus, value after 7 years $= 3000 \times 1.046^7 = \$4110.01$

(b) $5000 = 3000 \times 1.046^x \Rightarrow 1.046^x = \frac{5}{3} \Rightarrow x \ln(1.046) = \ln\left(\frac{5}{3}\right)$

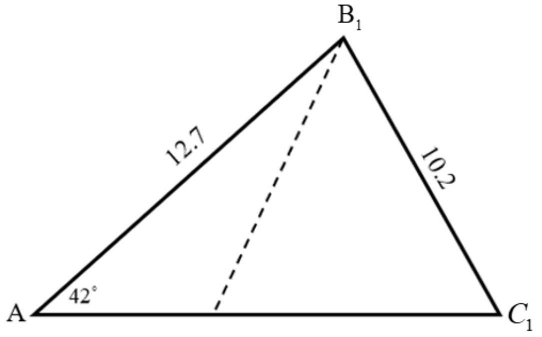
$$\Rightarrow x = \frac{\ln\left(\frac{5}{3}\right)}{\ln(1.046)} = 11.3584\ldots$$

The investment will exceed \$5000 after a minimum of 12 full years

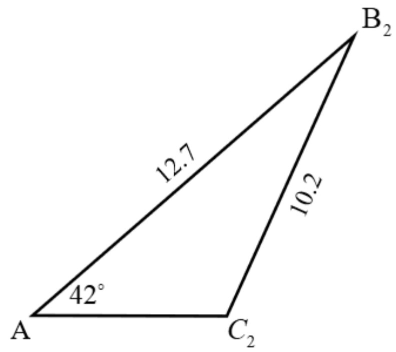
Hence, $x = 12$

SL Paper 2 Mock A 2020 - WORKED SOLUTIONS v1

3.



OR



$$\frac{\sin 42^\circ}{10.2} = \frac{\sin C_1}{12.7} \Rightarrow C_1 = \sin^{-1}\left(\frac{12.7\sin 42^\circ}{10.2}\right)$$

$$C_2 = 180^\circ - 56.422^\circ = 123.578^\circ$$

$$C_1 = 56.442^\circ \;\Rightarrow\; B_1 = 180^\circ - \left(56.422^\circ + 42^\circ\right)$$

$$B_2 = 180^\circ - \left(123.578^\circ + 42^\circ\right) = 14.422^\circ$$

$$B_1 = 81.578^\circ \;\Rightarrow\; \frac{\sin 81.578^\circ}{AC_1} = \frac{\sin 42^\circ}{10.2}$$

$$\frac{\sin 14.222^\circ}{AC_2} = \frac{\sin 42^\circ}{10.2}$$

$$AC_1 = \frac{10.2\sin 81.578^\circ}{\sin 42^\circ} = 15.079\,\text{cm}$$

$$AC_2 = \frac{10.2\sin 14.422^\circ}{\sin 42^\circ} = 3.7966\,\text{cm}$$

Hence, the two possible lengths of AC are 15.1 cm and 3.80 cm

4. general term of $(4x+p)^5$ is $\binom{5}{r}(4x)^{5-r}p^r$

exponent of x is $5-r$; for the x^3 term then $5-r=3 \;\Rightarrow\; r=2$

$$\binom{5}{2}(4x)^{5-2}p^2 = 10(4x)^3p^2 = 640p^2x^3;$$

hence, $640p^2 = 160 \;\Rightarrow\; p^2 = \dfrac{160}{640} = \dfrac{1}{4} \;\Rightarrow\; p = \pm\dfrac{1}{2}$

SL Paper 2 Mock A 2020 - WORKED SOLUTIONS v1

5. Let X be the random variable representing time (in minutes) it takes for a student to travel to school

$$\mathrm{P}(X < 5) = 0.04 \quad \Rightarrow \quad Z \approx -1.75069...$$

$$\mathrm{P}(X < 25) = 0.7 \quad \Rightarrow \quad Z \approx 0.524401...$$

Using formula for standardized normal variable $Z = \dfrac{x-\mu}{\sigma}$

$$-1.75069... = \frac{5-\mu}{\sigma} \quad \Rightarrow \quad \mu - 1.75069\sigma = 5$$

$$0.524401... = \frac{25-\mu}{\sigma} \quad \Rightarrow \quad \mu + 0.524401\sigma = 25$$

Solving system of linear equations: $\mu \approx 20.4\,\text{min}, \;\; \sigma \approx 8.79\,\text{min}$

6. $v(t) = \int a(t)\,\mathrm{d}t = \int\left(\dfrac{3}{t} + 2\sin 2t\right)\mathrm{d}t = 3\int\dfrac{1}{t}\,\mathrm{d}t + 2\int\sin 2t\,\mathrm{d}t$

$$\int\frac{1}{t}\,\mathrm{d}t = \ln t, \;\; \int\sin 2t\,\mathrm{d}t = -\frac{1}{2}\cos 2t$$

$$\Rightarrow \; v(t) = 3\ln t - \cos 2t$$

At $t = 1$, the particle is at rest, i.e. $v(1) = 0$, so

$$v(1) = 3\ln 1 - \cos 2(1) + C = 0$$

$$\Rightarrow \; C = \cos 2 = -0.4161\ldots$$

At $t = 6$:

$$v(6) \approx 3\ln 6 - \cos 2(6) - 0.4161 = 4.1153\ldots$$

Hence, $v(6) \approx 4.12 \;\; \mathrm{m\,s^{-1}}$

SL Paper 2 Mock A 2020 - WORKED SOLUTIONS v1

Section B

7. (a) Input data into GDC to determine the linear regression equation L_1:

$y = 10.7x + 121$ (values accurate to 3 significant figures)

(b) (i) gradient of regression equation is additional cost per box, i.e. **unit cost**

(ii) y-intercept of regression equation is the **fixed costs**, i.e. cost when zero boxes are produced

(c) $y = 10.6555(60) + 120.794 = 760.124$

Hence, cost of 60 boxes is approximately \$760

(d) $19.99x > y = 10.6555x + 120.794$

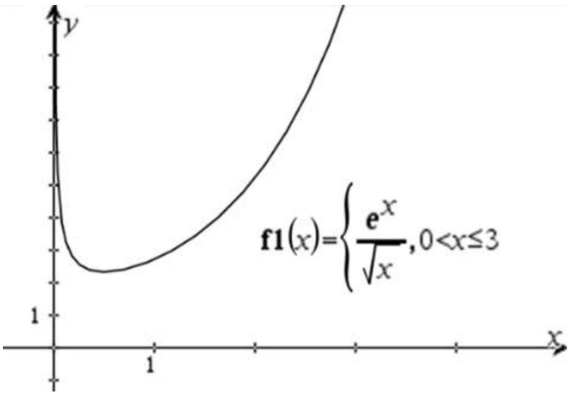
$\Rightarrow 9.3345x > 120.794 \Rightarrow x > 12.9405\ldots$

Hence, the factory must produce at least 13 boxes per day to make a profit

(e) This would be extrapolation, which is not appropriate

SL Paper 2 Mock A 2020 - WORKED SOLUTIONS v1

8. (a) (i)



(ii) $h(x)=\dfrac{e^x}{\sqrt{x}}=\dfrac{e^x}{x^{\frac{1}{2}}} \Rightarrow h^{-1}(x)=\dfrac{x^{\frac{1}{2}}e^x-\frac{1}{2}x^{-\frac{1}{2}}e^x}{\left(x^{\frac{1}{2}}\right)^2}=\dfrac{e^x\left(\sqrt{x}-\frac{1}{2\sqrt{x}}\right)}{x}=\dfrac{e^x\left(\frac{2x-1}{2\sqrt{x}}\right)}{x}=e^x\left(\dfrac{2x-1}{2x\sqrt{x}}\right)$

(iii) gradient of normal to curve is $-\dfrac{2x\sqrt{x}}{e^x(2x-1)}=\dfrac{2x\sqrt{x}}{e^x(1-2x)}$

(b) (i) gradient of (PQ) is $\dfrac{y-0}{x-1}=\dfrac{\frac{e^x}{\sqrt{x}}-0}{x-1}=\dfrac{e^x}{\sqrt{x}}\cdot\dfrac{1}{x-1}=\dfrac{e^x}{\sqrt{x}(x-1)}$

(ii) Equating the two expressions for gradient of normal to the curve gives

$\dfrac{e^x}{\sqrt{x}(x-1)}=\dfrac{2x\sqrt{x}}{e^x(1-2x)} \Rightarrow x\approx 0.545428...$ this is the x-coordinate of P

y-coordinate of P is $h(0.545428...)=\dfrac{e^x}{\sqrt{0.545428...}}\approx 2.33619...$

minimum distance from Q to graph of h is length of PQ

hence, minimum distance $=\sqrt{(0.545428...-1)^2+(2.33619...-0)^2}\approx 2.380001...$

minimum distance from Q to graph of h is approximately 2.38

SL Paper 2 Mock A 2020 - WORKED SOLUTIONS v1

9. (a) (i) Binomial distribution with $n=5$ and $p=\frac{1}{5}$:

$$\mathrm{E}(X)=np=5\cdot\frac{1}{5}=1$$

(ii) $\mathrm{P}(X\geq 3)=\mathrm{P}(X=3)+\mathrm{P}(X=4)+\mathrm{P}(X=5)$

$$\mathrm{P}(X=3)=\frac{5!}{(5-3)!3!}\cdot\left(\frac{1}{5}\right)^3\left(1-\frac{1}{5}\right)^{5-3}=\frac{5!}{2!3!}\cdot\frac{1}{125}\cdot\frac{16}{25}=\frac{32}{625}$$

$$\mathrm{P}(X=4)=\frac{5!}{1!4!}\cdot\frac{1}{625}\cdot\frac{4}{5}=\frac{4}{625}$$

$$\mathrm{P}(X=5)=\frac{5!}{5!}\cdot\frac{1}{3125}\cdot 1=\frac{1}{3125}$$

$$\Rightarrow\ \mathrm{P}(X\geq 3)=\frac{32}{625}+\frac{4}{625}+\frac{1}{3125}=\frac{181}{3125}=0.05792$$

(b) (i) $0.67+0.05+(a+2b)+(a-b)+(2a+b)+0.04=1$

$\Rightarrow\ 4a+2b=0.24$

(ii) $\mathrm{E}(Y)=\sum y\mathrm{P}(Y=y)=1$

$\Rightarrow\ 0\cdot 0.67+1\cdot 0.05+2(a+2b)+3(a-b)+4(2a+b)+5\cdot 0.04=1$

$\Rightarrow\ 13a+5b=0.75$

From (i): $4a+2b=0.24\ \Rightarrow\ b=0.12-2a$

$\Rightarrow\ 13a+5(0.12-2a)=0.75\ \Rightarrow\ 3a+0.6=0.75$

$\Rightarrow\ 3a=0.15\ \Rightarrow\ a=0.05$

Substituting back into equation from (i):

$b=0.12-2(0.05)\ \Rightarrow\ b=0.02$

(c) $\mathrm{P}(Y\geq 3)=\mathrm{P}(Y=3)+\mathrm{P}(Y=4)+\mathrm{P}(Y=5)=0.03+0.12+0.04=0.19>0.05792$

Hence, Isabel is more likely to pass the test.